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ABSTRACT

A new method for the measurement of consensus in small groups is proposed. The method allows for the study of the dynamic formation of consensus and, with the aid of some mathematical manipulations, reveals the development of subgroup relations over time. In a pilot test, the method was used to study groups of students choosing a topic which they would later research for presentation to the class as a whole. Results using the new measurement technique indicate that coalitions are fluid and fragile; that an individual may be a member of more than one coalition at the same time; that the strength of coalitions increases as the groups move toward consensus; and that conceptualizing consensus as unanimous agreement on a final topic oversimplifies the depth and complexity of the communication process involved in reaching the consensus outcome. (Author/AA)

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A DYNAMIC MEASUREMENT OF CONSENSUS IN SMALL DECISION MAKING GROUPS

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A DYNAMIC MEASUREMENT OF CONSENSUS

SMALL DECISION MAKING GROUPS

Abstract

A new method for the measurement of consensus in small groups is proposed. The method allows for the study of the dynamic formation of consensus and with the aid of some mathematical manipulations reveals the development of subgroup relations over time. The method is applied in a pilot test to several groups and the results are reported.

A DYNAMIC MEASUREMENT OF CONSENSUS IN SMALL DECISION MAKING GROUPS

I. Introduction

During the last forty years, small group researchers have been concerned with studying consensus in small decision making groups. It has been generally recognized that consensus is one of the major goals of a group discussion (Gouran 1974, Kline 1972, Knutson 1972, Kowitz and Knutson 1975). In fact, in terms of the algebraic model of small group communication suggested by Spillman and Spillman (1976), consensus is one of the major elements of the small group output set. Yet, current procedures for the experimental measurement of the degree of consensus achieved in a given small group do not allow for an adequate theoretical development of the concept.

Consensus has been generally defined as unanimous agreement. However, since groups do not usually reach complete consensus, experimental and theoretical problems have arisen when attempting to measure "distance" from consensus. Hare (1952) gave groups a rank ordering task, and defined consensus as the difference between their average correlation of all rank orders before the discussion and their average correlation after the discussion. Godwin and Restle (1974) had groups choose between four alternative solutions to a given problem. Group members were assumed to prefer only one of the alternatives at any given point in time and consensus was reached only when all the group members preferred the same alternative. No measure of "distance" from consensus was suggested in the Godwin and Restle (1974) study. Kline (1972) gave a group six alternative choices ranging from the most conservative policy to the most liberal. The distance from consensus was determined at the conclusion of the discussion by the total number of positions subjects were away from the most agreed upon

policy. Each alternative was assumed to be equidistant from its adjacent alternatives. Each group member was assumed to prefer only one of the six alternatives. Knutson (1972) used Kline's procedure using five alternative policies ranging from the most conservative to the most liberal. Knutson and Holdridge (1975), Kowitz and Knutson (1975), and Barnes (1975) all attempted to measure the perceived consensus of a small group at the conclusion of its discussion by administering a six item 7-point Likert-type Preceived Consensus Test developed by Knutson, Lee and Danes, 1973. The amount of consensus is calculated by summing up the ratings for all the group members. Finally, Hill (1976) measured consensus by asking individual group members to rate their own level of agreement with the final group decision.

A number of theoretical and empirical problems exist with all these measures. For example, in some of the measures the groups are constrained to select one of several preselected alternatives. In addition it is assumed that each group member prefers only one alternative during disucssion. These alternatives are also considered to be equidistant on some scale. Most importantly, however, these measures consider consensus as a static concept. They measure consensus at the completion of the group discussion. This is inconsistent with the recent concern for studying communication in small groups as an on-going process. If small groups are to be considered from a general systems perspective, the dynamic structure of small group variables has to be studied. Hence, our measurement procedures must reflect the dynamic nature of these variables. Spillman and Spillman (1976) indicated: "Central to the idea of process and included in almost every definition of the term is the concept of change over time. Hence, any process-oriented study of small group behavior should consider dynamic constructs."

In this paper, a new procedure for the dynamic measurement of consensus in small groups is suggested. To illustrate its use, the measure is applied to a

sample of four groups. The structure revealed by the measure is analyzized and future research possibilities using this measure are outlined.

II. A Dynamic Measurement of Consensus

The proposed measurement instrument is based on the notion of fuzzy preference orderings for individuals and groups using the theory of fuzzy relations developed by Zadeh (1965). The concept of a fuzzy relation or a fuzzy set is basically simple. In classical set theory, given a set A and an element a_i, then a_i is either a member of the set A or it is not. This is represented by the characteristic function of the set A given by:

$$u_A(a_i) = \begin{cases} 1 & \text{if } a_i \text{ is a member of the set A} \\ 0 & \text{if } a_i \text{ is not a member of the set A} \end{cases}$$

Thus, the characteristic function of a conventional set has two possible values, 0 and 1. Fuzzy sets generalize the notion of a characteristic function, allowing it to assume any value between zero and one, so that an element a_i could be a partial member of the fuzzy set A^1. In other words, it could posses some but not necessarily all of the characteristics of the prototypical members of the set A^1. Blin (1974) first suggested applying this concept to group decision theory. The application of fuzzy sets to the measurement of consensus suggested in this paper represents a significant improvement over Blin's approach.

A small decision making group considers a set of alternative solutions, A. A fuzzy preference relation is defined over pairs of alternatives (a_i, a_j) in AxA as follows:

$$p(a_i,a_j) = \begin{cases} 1 & \text{if alternative } a_i \text{ is definitely preferred over } a_j \\ .5 & \text{if there is no preference between } a_i \text{ and } a_j \\ 0 & \text{if alternative } a_j \text{ is definitely preferred over } a_i \end{cases}$$

If $p(a_i,a_j)$ is between .5 and 1 it indicates a slight preference for a_i over a_j and if it is between 0 and .5 it indicates a slight preference for a_j over a_i.

(We can write $p(a_i,a_j)$ as $(a_i \rightarrow a_j)$ to indicate that the measure is of how much a_i is preferred to a_j). This fuzzy preference relation can be represented by a fuzzy relation matrix, where the ij-th element of the matrix is given by:

$$r_{ij} = p(a_i,a_j)$$

Relation matrices of this type can represent preference assignments of an individual or a group. This fuzzy relation matrix, R, has the following properties:

(1a) $r_{ij} = 0$

(1b) $r_{ij} + r_{ji} = 1$

Property (1a) indicates that given a choice between alternative a_i and itself, alternative a_i, of course, will always be choosen. Using zero instead of one will simplify subsequent manipulation without affecting the intended meaning. Property (1b) indicates that the preference for a_i over a_j is one minus the preference for a_j over a_i. Because of property (1b), the fuzzy relation matrix, R, will be called a recipocal fuzzy relation. An example of a recipocal fuzzy relation matrix is given by the 3 x 3 matrix below:

$$R = \begin{array}{c} \\ a_1 \\ a_2 \\ a_3 \end{array} \begin{array}{c} \begin{array}{ccc} a_1 & a_2 & a_3 \end{array} \\ \begin{bmatrix} 0 & .9 & .2 \\ .1 & 0 & .7 \\ .8 & .3 & 0 \end{bmatrix} \end{array}$$

It can be inferred from this matrix, for example, that alternative a_2 is preferred over a_1 by .9, hence a_1 is preferred over a_2 by .1.

Given a recipprocal fuzzy relation matrix, R, a pair of scalar measures can be calculated which are complimentary in the sense that as one increases the other decreases. They will be called the fuzziness of the relation, F(R), and the consensus of the relation, C(R). For a n x n matrix (that is, n alternatives) they are defined as:

$$(2a) \quad F(R) = \frac{2tr(R^2)}{n(n-1)}$$

$$(2b) \quad C(R) = \frac{2tr(RR^t)}{n(n-1)}$$

In equation (2), Tr(B) is the trace of the matrix B (the sum of the diagonal elements) and B^t is the transpose of the matrix B (the rows and columns of B are interchanged). An examination of the components of these measures reveals the motivation for their use: The trace of R^2 consists of the sum of terms $(r_{ij})(r_{ji})$ which are of the form $(a_i \rightarrow a_j)(a_i \leftarrow a_j)$. This is proportional to the relative indecisiveness or fuzziness generated by attempting to choose between alternatives a_i and a_j. Since, there are precisely $n(n-1)/2$ pairs of alternatives (a_i, a_j) with $i \neq j$, F(R) measures the average amount of fuzziness over all possible alternatives. Similarly, the trace of RR^t consists of the sum of terms $(r_{ij})(r_{ij})$ which are of the form $(a_i \rightarrow a_j)(a_i \rightarrow a_j)$, which measures the degree to which a_i dominates a_j. For the 3 x 3 matrix F(R) = .31 and C(R) = .69.

The fuzziness measure, F(R), is an indication of overall indecision in the rankings for a given matrix. F(R) assumes its maximum value when the matrix R contains the value 1/2 in all the off-diagonal elements. In this case, all alternatives are equally preferable. On the other hand, F(R) assumes its minimum value when the matrix R contains all zero's and one's. In which case, the individual has a definite preference for each alternative pair.

The following theorem relating F(R) and C(R) is stated without proof, interested readers may consult Bezdek, Spillman and Spillman (1977) for proof:

Theorem one: Let $n \geq 1$ then

$$(3a) \quad 0 \leq F(R) \leq 1/2$$

$$(3b) \quad 1/2 \leq C(R) \leq 1$$

$$(3c) \quad F(R) + C(R) = 1$$

$$(3d) \quad F(R) = 0 \longleftrightarrow C(R) = 1$$

$$(3e) \quad F(R) = 1/2 \longleftrightarrow C(R) = 1/2$$

Theorem one indicates that when fuzziness is at its maximum, consensus is at its minimum, and conversely, when fuzziness is at its minimum, consensus is at its maximum. In applying thie model the group has reached consensus when $C(R) = 1$ (or $F(R) = 0$). This is true only if the group task is to rank order all the alternatives. In this case the group must consider each alternative and its relation to all the other alternatives. If consensus is achieved, the final matrix will consist of zero's and one's, implying that $C(R) = 1$. However, if the groups task is to select one alternative from among several possible alternatives, certain alternatives will drop from the groups consideration.

Preference's between two alternative's which are no longer considered by the group will be .5 (this was found to be the case in the pilot study reported later in this paper). The value of C(R) for this task situation is not one when the group reaches consensus, rather it is .4, as calculated from the following matrix, R, where the i-th alternative was selected:

$$R^* = \text{i-th row} \quad \left|\begin{array}{ccccc} 0 & & 1 & 1/2 & \\ & 0 & 1 & & \\ 0 & 0 & 0 & 0 & 0 \\ & 1/2 & 1 & 0 & \\ & & 1 & & 0 \end{array}\right|$$

This matrix indicates that the preference for the selected alternative over all other alternatives is one or zero (depending on whether the selected alternative is the first or second one in the comparison pair, and the preference relation between all other pairs is .5. The measure of consensus for this nxn matrix is given by:

$$(4a) \quad C(R^*) = (n + 2)/(2n)$$

and the measure of fuzziness for this matrix is:

$$(4b) \quad F(R^*) = (n - 2)/(2n)$$

These will be called the base consensus C_B and base fuzziness F_B. For example, if the group were required to select one alternative from ten possible alternatives (n = 10), consensus might be reached only if $C(R) = .6 = C_B$ and $F(R) = .4 = F_B$. Given a final group matrix, R' with a consensus measure C(R'), the distance of the group from consensus is found from:

$$(5) \quad d(C) = \left| \frac{(n+2)}{2n} - C(R') \right|$$

For example, if n = 10, $d(C) = |.6 - C(R')|$. For the 3 x 3 matrix used earlier consensus would be indicated by a $C_B = .833$, hence, the distance from consensus for that matrix is $d(C) = |.833 - .69| = .1433$. The range of C(R) as given by theorem one is 1/2 to 1. In order to better interpret the meaning of d(C), the range of C(R) can be normalized to between 0 and 1 using the transformation:

$$(6) \quad C_n(R) = 2C(R)-1$$

where $C_n(R)$ is the normalized consensus value. In terms of the normalized consensus value the distance from consensus, d(C) is given by:

$$(7) \quad d(C) = 1/2 \quad |C_{bn} - C_n(R)|$$

In a similar manner, d(C) can be normalized to range between 0 and 1 using the transformation:

$$(8) \quad d_n(C) = d(C)/\max [|C_B-1| , |C_B-1/2|]$$

For the 3x3 matrix, the normalized distance from consensus is $d_n(C) = .43$ indicating that the group is almost half way to consensus.

α-Level Sets

There is a great deal of information contained in each reciprocal fuzzy preference matrix. If the matrix is large, it could be difficult to locate the important details by inspection. This motivates the development of α-level sets. An α-level set is defined as the set of all pairs (a_i,a_j) whose preference level is greater than or equal to α. The formal definition is:

$$S = \left[(a_i,a_j) \mid u_A(a_i,a_j) \geq \alpha\right]$$

These α-level sets can also be represented by a relation matrix. The α-level relation matrix is constructed from the fuzzy matrix by placing a 1 in the ij-th position of the α-level if the value of the ij-th position of the fuzzy matrix is greater than or equal to α. For the 3x3 fuzzy relation matrix, α-level matrices are given below:

$\alpha = .9$ $\begin{bmatrix} 0 & 1 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}$ $\alpha = .8$ $\begin{bmatrix} 0 & 1 & 0 \\ 0 & 0 & 0 \\ 1 & 0 & 0 \end{bmatrix}$ $\alpha = .7$ $\begin{bmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ 1 & 0 & 0 \end{bmatrix}$ $\alpha = .3$ $\begin{bmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ 1 & 1 & 0 \end{bmatrix}$ etc.

The α-level matrices are non-fuzzy or "hard" matrices since they contain just zero's and one's. These matrices indicate more clearly than do the fuzzy matrices which alternatives are preferred at the given α-level. If a column of an α-level matrix contains all one's except on the diagonal (the diagonal of an α-level matrix for $\alpha > 0$ is always 0 by property (1a) of the original fuzzy relation matrix) then the alternative represented by that column is definitely preferred over all the other alternatives at the α-level. For example matrix, alternative 2 is preferred over all other alternatives at the .3 α-level.

In addition to providing information on the preference structure of the group, α-level matrices could be used to identify subgroup membership, as well as measuring the strength of subgroup membership and the dynamic evolution of subgroup formation in decision making. Traditionally, two group members are considered to be in a subgroup if their highest ranked alternatives are the same. This procedure for determining subgroup membership does not take into account the individuals feelings on all other alternatives, they could be radically different and yet they would be ranked within the same subgroup. Comparisons of various α-level relation matrices determine subgroup membership based on each members rankings for all the alternatives. For example, given the two .9 α-level relation matrices for members one and two below

notice that for every one in the matrix M_1 there is a one in matrix M_2. These two member agree on every alternative that they have ranked .9 or above. They could be said to be in a subgroup at the .9 α-level.

$$M_1 = \begin{bmatrix} 0 & 0 & 1 \\ 0 & 0 & 0 \\ 0 & 1 & 0 \end{bmatrix} \qquad M_2 = \begin{bmatrix} 0 & 0 & 1 \\ 0 & 0 & 0 \\ 0 & 1 & 0 \end{bmatrix}$$

In general, a method to determine if two group members are part of the same subgroup at a given α-level is to count the number of ones they have in common in their α-level relation matrix and divide that number by the total number of one's in the two matrices. If M_1 is the α-level relation matrix of member one and M_2 is the α-level relation matrix of member two and count (M_1) is the number of one's in M_1 and count(M_2) is the number of one's in M_2, the level of agreement, at the α-level between members one and two is:

$$(9) \quad A_\alpha = \frac{2\,tr(M_1 M_2^+)}{(\text{count}(M_1) + \text{count}(M_2))}$$

Where $tr(M_1 M_2^+)$ gives the number of one's M_1 and M_2 have in common. A_α, the level of agreement, has a maximum value of one if M_1 and M_2 have identical preferences and a minimum value of 0 if M_1 and M_2 have no one's in common. In the previous example, the level of agreement was one, in the example below the level of agreement is .5

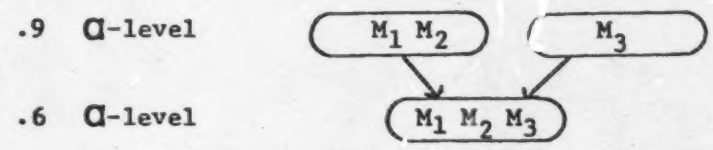
$$M_1 = \begin{array}{c} \\ a_1 \\ a_2 \\ a_3 \end{array} \begin{array}{c} \begin{array}{ccc} a_1 & a_2 & a_3 \end{array} \\ \begin{bmatrix} 0 & 0 & 1 \\ 1 & 0 & 1 \\ 0 & 1 & 0 \end{bmatrix} \end{array} \qquad M_2 = \begin{array}{c} \\ a_1 \\ a_2 \\ a_3 \end{array} \begin{array}{c} \begin{array}{ccc} a_1 & a_2 & a_3 \end{array} \\ \begin{bmatrix} 0 & 1 & 1 \\ 0 & 0 & 1 \\ 1 & 0 & 0 \end{bmatrix} \end{array}$$

An MxM subgroup membership matrix can be calculated for each level of agreement, A_α, where M is the number of members in the group. The ij-th element of this matrix, is the α-level at which M_i and M_j have a level of agreement A_α. For example, given the subgroup membership matrix for the .8 level of agreement,

$SG_{.8}$, below:

$$SG_{.8} = \begin{array}{c} \\ M_1 \\ M_2 \\ M_3 \end{array} \begin{array}{c} \begin{array}{ccc} M_1 & M_2 & M_3 \end{array} \\ \begin{bmatrix} 1 & .9 & .6 \\ .9 & 1 & .6 \\ .6 & .6 & 1 \end{bmatrix} \end{array}$$

M_1 and M_2 have a level of agreement of .8 at the .9 α-level (80% of their alternative rankings of .9 and above are the same) and M_2 and M_3 have a level of agreement of .8 at the .6 α-level (80% of their alternative rankings of .6 and above are the same). A graph of the subgroup membership corresponding to the matrix above would be:



An experiment to test this model would require the group members to construct a fuzzy relation matrix at various points in time. These matrices could then be analyzed to find the consensus measure and the subgroup relations. A computer program has been written which, given the lower half of the reciprocal fuzzy relation matrix (the upper half is not necessary since it equals one minus the lower half and can easily be calculated by the computer) will conduct a complete analysis of the group's decision making over time. Results of an experimental use of this procedure are reported in the next section.

III. Method

Subjects

Students in a fundamental speech communication course participated in the experiment as part of their regular group assignment. Four groups of 3, 4, 5 and 5 members were included. Members chose their groups voluntarily. Thus,

some groups members knew each other well while others were only slightly acquainted.

Task

The class assignment was for students to choose a topic and research it as thoroughly as possible for presentation to the class as a whole. The experimental task was for the groups to brain storm for a series of topics and to eliminate the less preferred topics and select one topic for presentation.

Procedure

At the beginning of the class E explained the general purpose of the experiment and the testing procedure. They were told to use brain storming procedures to create a list of no more than ten possible topics for presentation to the class. Though ten was a maximum, groups were told they could include fewer than ten in their final list of alternatives. Ten was set as the maximum, however, because of the time and complexity involved in filling out a preference matrix larger than 10x10.

Groups were divided and experimental assistants accompanied each group to a separate discussion room. The assistants observed, but did not take part in the discussion. After the groups had come up with their list of alternatives, the assistants handed out the preference scales and answered any questions related to the procedure for filling them out. After the first measurement time, the assistants, four senior and graduate students from a graduate course in small group discussion, administered the preference scales every five minutes until the groups had reached a final decision. The groups filled out the scale one more time at the completion of their discussion.

Each group took approximately three to five minutes to fill out the preference scale the first time period. After becoming familiar with the

instrument, however, the measurement time decreased to approximately one to two minutes. Groups took from 15 to 50 minutes to come up with their initial list of alternatives. One of the groups completed the experimental task in one day and filled out the preference scale three times. The other three groups continued the task on the second day, and these groups where measured three, five and five times respectively.

Alternative Preference Scale

As soon as the groups had chosen their ten alternative topics, the assistant placed the names of each topic in a 10x10 matrix. Participants were told to rate their personal preference for the topic labeled in the column over the topic labeled in the row for each square of the bottom triangle of the matrix, excluding the diagonal. This measure thus indicated the participants relative preference for every one of their alternatives over all other alternatives. Participants were allowed to fill in any number between zero and one in the matrix squares. As indicated earlier, a number between .51 and 1.0 indicated a relative preference for the column over the row, a .50 indicated no preference, and a score between .49 and 0.0 indicated a relative preference for the row over the column for that particular square.

IV. Analysis of the Consensus Measuring Instrument

Group and individual scores from preference scales were analyzed to determine their fuzziness, distance from consensus, and level of agreement at specified alpha levels at both a static and dynamic level. Results were intended to be descriptive, and future research will be designed to devise statistical and predictive equations for small group decision making. Recognizing that the scope of the group task was small and the measurement and discussion procedures were somewhat structured, the consensus indicators still provided a dynamic and provocative look at communication.

Consensus Formation

Several mathematical procedures were used to determine the groups' distance from consensus. First, to determine fuzziness at the individual and group level, participants filled out preference scales of their ten chosen alternatives (see Figures 1-4). By averaging individual fuzziness matricies, a group fuzziness matrix was constructed at each time period (see Figure 5). Using formula (2), the fuzziness and consensus measures were computed for each group matrix over time (see Table 1). Using formula (5), the distance from consensus was calculated and normalized using formula (7) for each matrix (see Table 2).

All groups were relatively close to consensus at the end of their discussion. Final measures ranged from .003-.218. Although the Knutson perceived consensus scale does not allow for an absolute measure of consensus, results from this scale were consistent with those obtained from our calculations (see Table 2). A dynamic analysis of consensus revealed an interesting movement toward and away from consensus over time (see Figure 6). In all groups, the distance from consensus increased during the first five minutes of discussion.

Three of the four groups (2, 3 and 4) moved back toward consensus at the end of the discussion. One possible explanation for this is that rather than eliminating topics from consideration immediately, several topics seem to increase in relative preference to other topics between the first and second measures.

An indication of the number of topics under consideration can be determined by adding the number of ones in each column of the α-level matrices. For group three, for example, at the .9 alpha level, the column sums are: 1, 0, 2, 0, 0, 1, 0, 1, 0, 0) at the time zero. At time one the column sums are: (3, 0, 4, 0, 0, 2, 0, 1, 0, 0). Finally, at time two the column sums reduce to (9, 0, 0, 0, 0, 0, 0, 0, 0, 0), (see Figure 7).

Coalition Formation

Using the procedure outlined in section two for determining subgroup membership, the data indicated a dynamic process of coalition development splitting, and reformation. Dynamic subgroup formation was calculated for all four groups (see Figures 8-11). The .8 level of agreement was used throughout the analysis. For group one at time zero, member two and three formed a coalition at the .4 alpha level; the structure was identical at time one. At time two, a strong coalition between numbers 1, 3, and 4 was formed at the 1 α-level. For group two at time zero, members one and three coalesced at the .5 alpha level; though structurally the same, the coalition was weaker at time one, it had reduced to the .4 alpha level. The next structural change occurred at time three where the member one and three coalition split and member one formed a new coalition with member two at the .5 alpha level. This structure was maintained throughout the rest of the discussion. For group three at time zero, members four and five coalesced at the .5 alpha level. Member four split from the original subgroup and formed a new one with member one at the 1.0 alpha level. At time two members three and five joined members one and four, leaving member two isolated at the 1 alpha level. Finally, a high degree of movement and complexity was exhibited in group four. At time zero, two coalitions were formed; a coalition between members one and three, and another between members two, three, four and five at the .5 alpha level. At time one the entire structure changed resulting in a single coalition between members one and four at the .5 alpha level. Another radical structural change occurred at time two with two new coalitions being formed; one between members two and three and another between members two and four at the 1.0 alpha level. At time three a single large coalition was developed between members two, three, four and five at the 1.0 alpha level. Finally, at time four member one was assimilated into the previously formed subgroup at the 1.0 alpha level.

Several implications about subgroup development are indicated by this preliminary data. First, coalitions are fluid and fragile. It would be beyond the scope of this paper to speculate the precise reasons for coalition shifts, but future suggestions for research will be described at the end of this paper. Second, coalitions are not transitive in nature. That is, an individual may be a member of more than one coalition at the same time (such as was exhibited in group four at times zero and two). Third, the general trend is for the strength of the coalitions, i.e., as indicated by increasing alpha levels, to increase as the groups move toward consensus. This was especially evident in groups one, three and four. Finally, conceptualizing and measuring consensus as unanimous agreement on a final topic or solution oversimplifies the complexity and depth of the communication process involved in reaching and, in fact, defining the final consensus outcome. Using a scaling technique as has been outlined in this paper may well lead to better predictions concerning later subgroup formation, individual commitment to the final decision, and group satisfaction with their task.

Implications for Future Research

This new dynamic measure of consensus offers several interesting avenues for future research and conceptualization. First, adjustments in the measuring instrument should be made to cover broader and more complex decision making situations. Second, replications of previous consensus studies should be undertaken using this more sophisticated measuring instrument. Third, the complex relationship between member satisfaction, distance from consensus, and the final structural dynamics of the group should be studied thoroughly to understand better the process of consensus formation and the quality and quantity of group productivity. Fourth, comprehensive studies of the relation between communicative

interaction, coalition formation and consensus development should be undertaken so that descriptions and predictions of the entire process of group communication can be formulated. Questions from this research perspective could include: (1) what are the communicative patterns related to coalition formation, coalition splitting, and communication between coalitions and isolated members; (2) what communicative patterns characterize communication between competing coalitions; (3) what communicative patterns differeniate individuals with highly fuzzy preference matrices as opposed to those with increasingly less fuzzy matrices; (4) how are communicative patterns different between groups which reach consensus early in their disucssion as opposed to groups who reach consensus later.

Summary

Gouran (1974) called strongly for an emphasis on decision making and consensus formation in communicative research. He, and others have argued that decision making provides a unique arena in which communication researchers may add to the understanding of man. Pragmatically, he also argued that a study of decision making and consensus is vital to the functioning of every day society. This paper has described the mathematics and conceptualization behind a new method of measuring consensus in small groups. The measure has the advantages of being dynamic, it considers the entire alternative structure of individuals and groups rather than a single topic or decision under discussion, and it allows for a clear description of group structured fluctuations over time.

References

Barnes, J.A., A process analysis of the message structure of consensus and nonconsensus decision making groups, unpublished Ph. D. disseration, University of Southern California, 1975.

Bezdek, J., Spillman, B.M., and Spillman, R.S., "Fuzzy measures of uncertainty and consensus for group decision," in review for 1977 IEEE Prodeedings on Decision and Control.

Blin, J.M., "Fuzzy relations in group decision theory," J. of Cybernetics, 1974, p. 17-22.

Goodwin, W.F. and Restle, F., "The road to agreement: Subgroup pressures in small group consensus processes," J. of Personality and Social Psychology, 1974, p. 500-509.

Gouran, D.S., Discussion: The process of group decision-making. New York: Harper and Row, 1974.

Hare, A.P., "A study of interaction and consensus in different sized groups," Am. Soc. Review, 1952, p. 261-267.

Hill, T.A. "An experimental study of the relationship between opinionated leadership and small group consensus," Communication Monographs, 1976, p. 246-257.

Kline, J.A., "Orientation and group consensus," Central States Speech J., 1972, p. 44-47.

Knutson, T.J., "An experimental study of the effects of orientation behavior on small group consensus," Speech Monographs, 1972, p. 159-165.

Knutson, T.J., Lee, J., and Danes, J.E., "Construction of a test to measure perceived consensus in small groups," presented at sca, New York City, 1973.

Knutson, T.J. and Holdridge, W.E., "Orientation behavior, leadership, and consensus: A possible functional relationship," Speech Monographs, 1975, p. 107-114.

Kowitz, A.C. and Knutson, T.J., "Effects of information type and orientation on consensus-achievement in substantive and affective small-group conflict." presented at W.S.C.A., Seattle, November 1975.

Spillman, R.J. and Spillman, B.M., "An algebraic model of small group communication: An open systems perspective," presented at ICA, Portland, April 1976.

Spillman, R.J., and Spillman, B.M., "The dynamic structure of small group behavior: Methods of analyzing change over time in Markov models," presented at W.S.C.A., San Fransico, November, 1976.

Zadeh, L., "Fuzzy sets," Information and Control, 1965, p. 338-353.

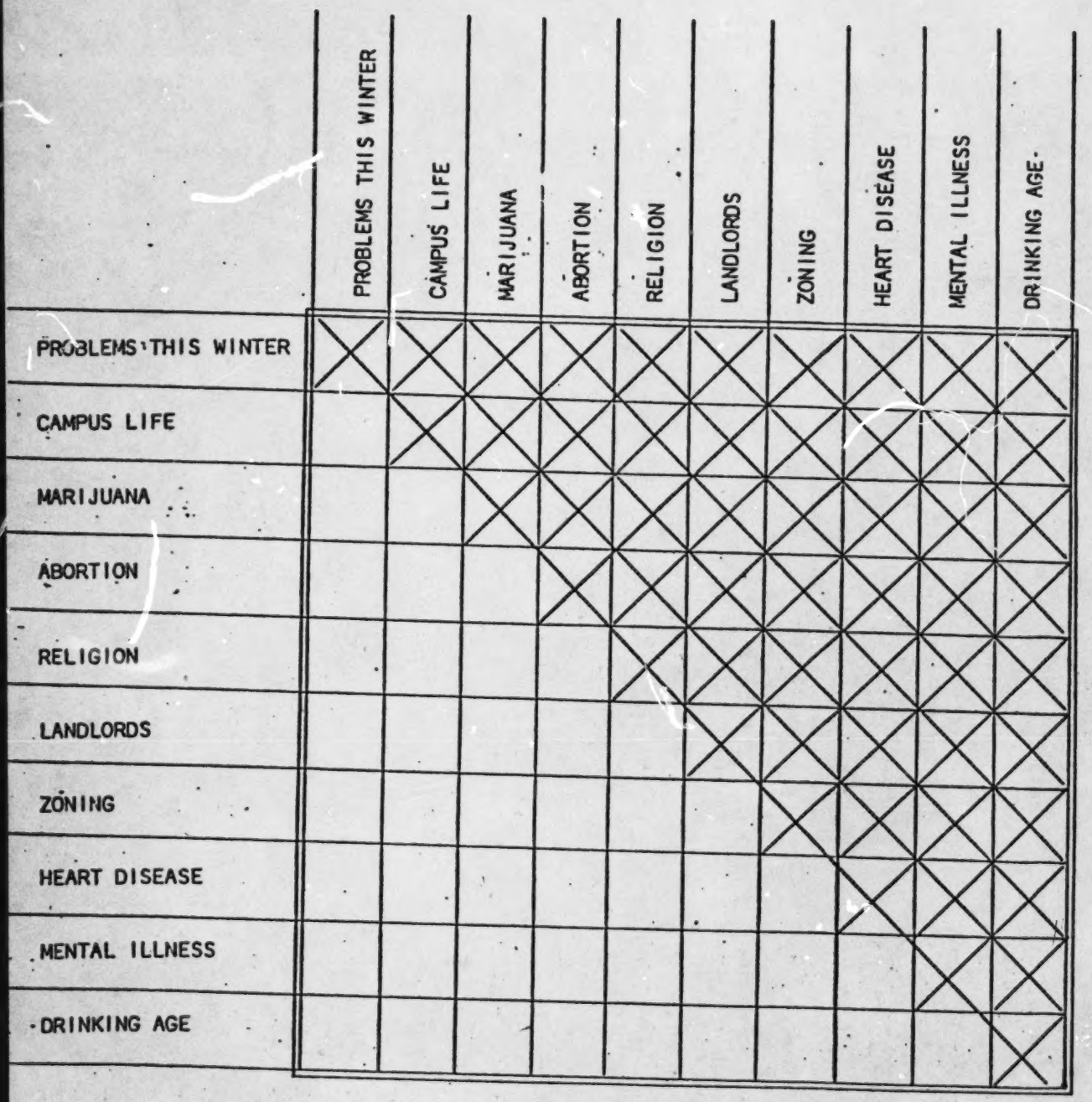


Figure 1: Measuring Instrument for Group One

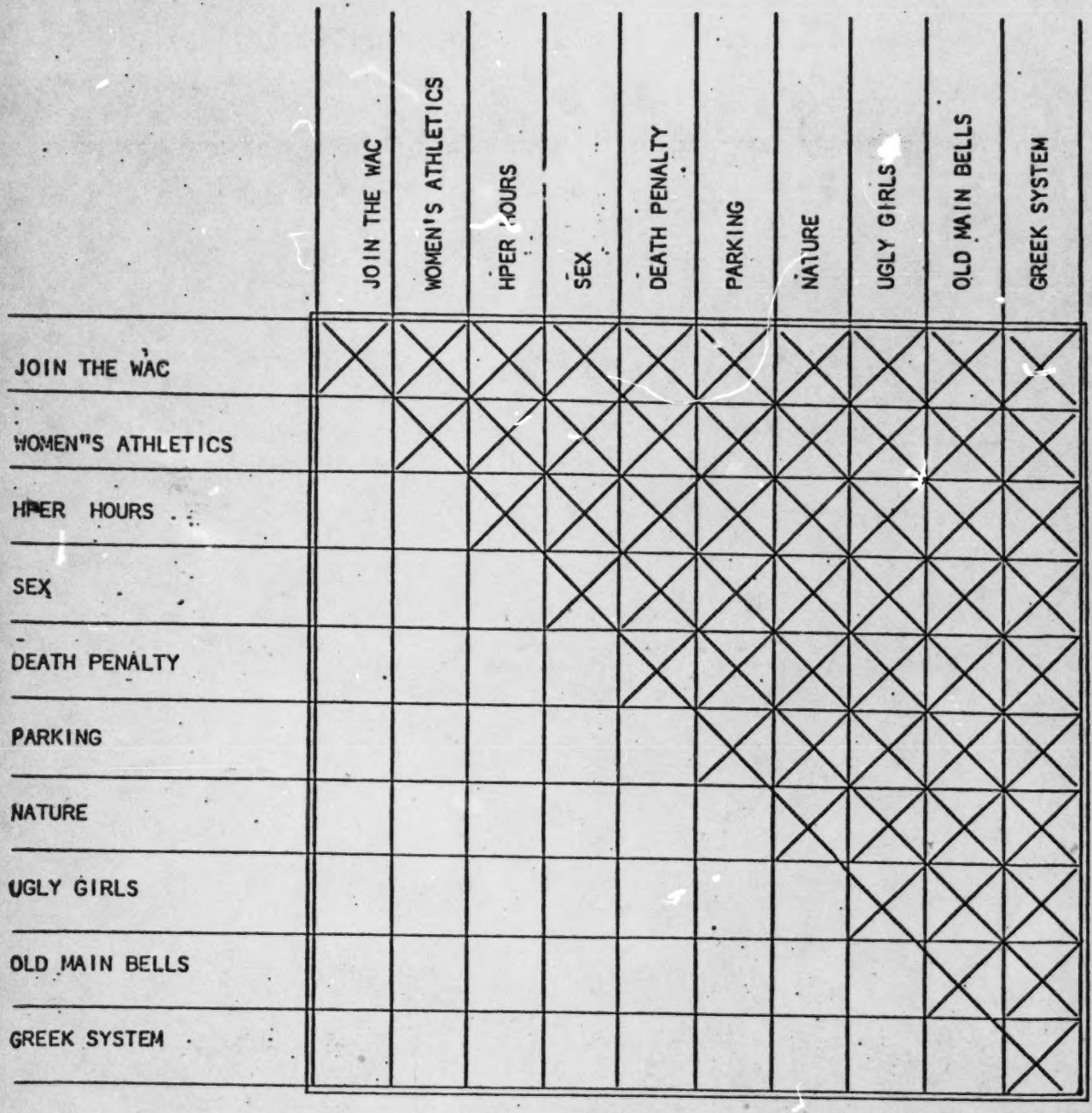


Figure 2: Measuring Instrument for Group Two

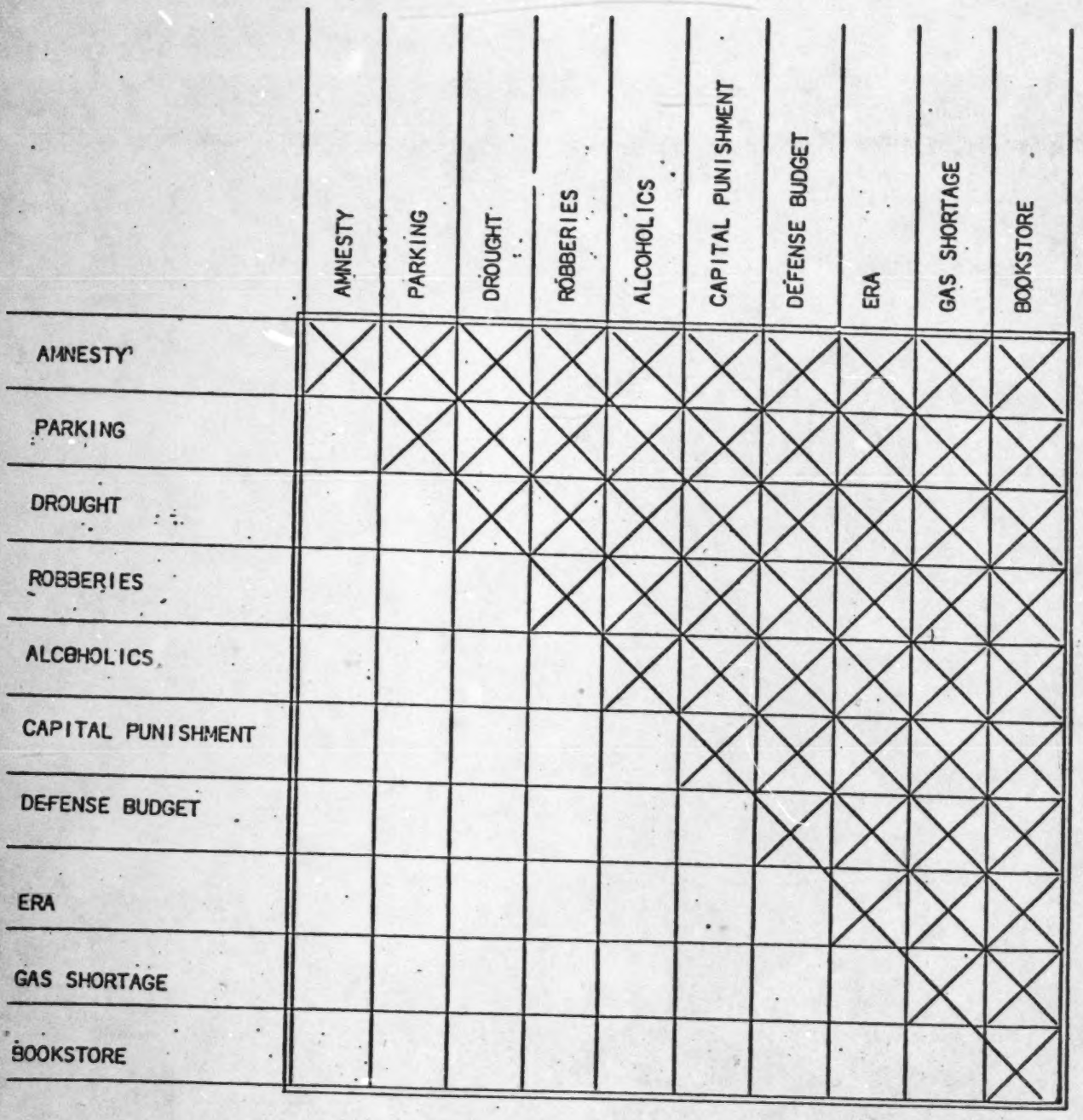


Figure 3: Measuring Instrument for Group Three

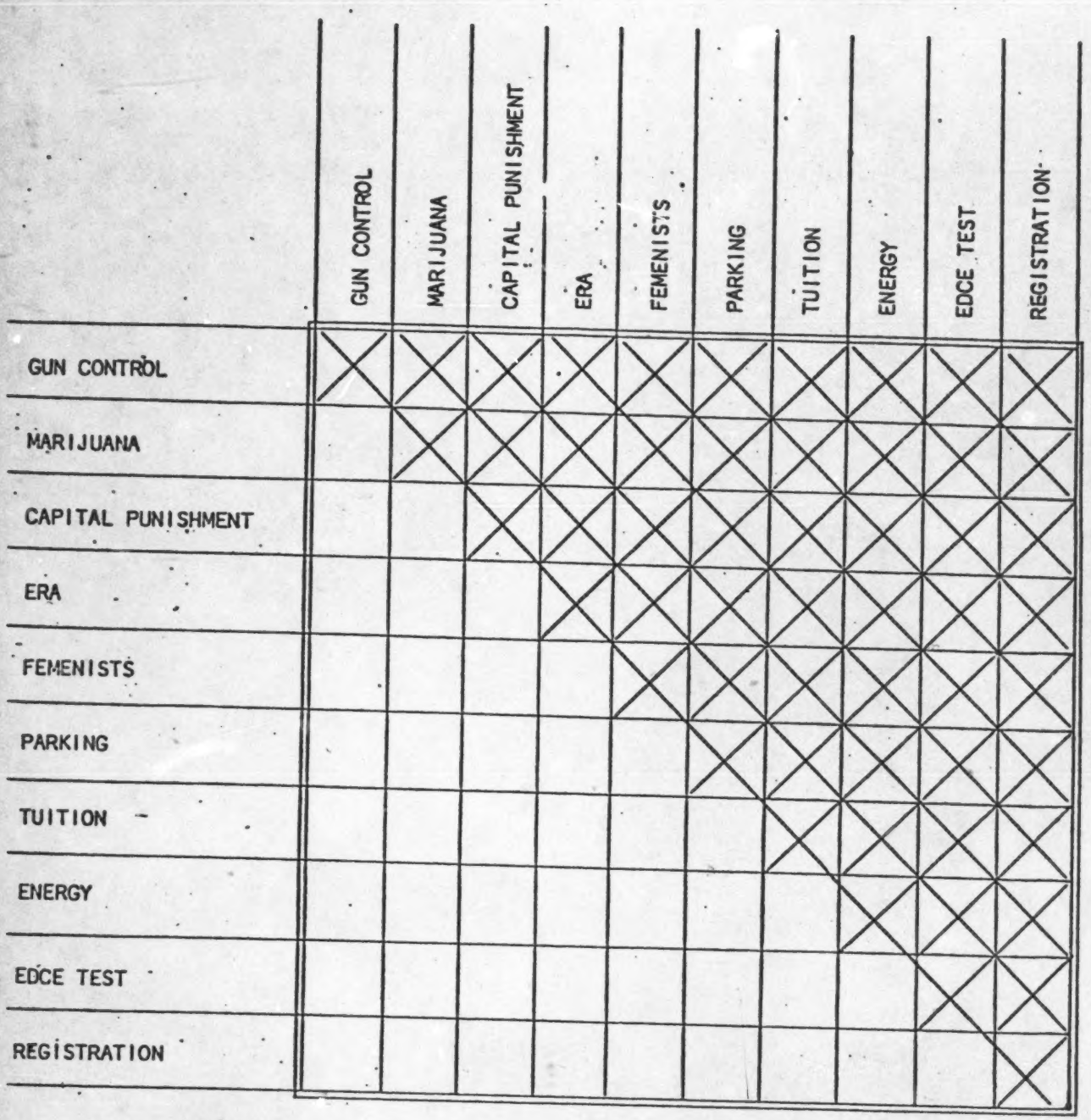


Figure 4: Measuring Instrument for Group Four

GROUP ONE

0	.45	.45	.45	.55	.45	.45	.5	.9	.48
.55	0	.53	.53	.55	.48	.45	.53	.9	.53
.55	.48	0	.5	.55	.48	.45	.53	.9	.45
.55	.48	.5	0	.55	.43	.45	.53	.9	.5
.45	.45	.45	.45	0	.45	.43	.45	.85	.48
.55	.53	.53	.58	.55	0	.5	.53	.9	.53
.55	.55	.55	.55	.58	.5	0	.53	.9	.53
.5	.48	.48	.48	.55	.48	.48	0	.88	.48
.1	.1	.1	.1	.15	.1	.1	.13	0	.08
.53	.48	.55	.5	.53	.48	.48	.53	.93	0

GROUP TWO

0	.17	.17	.33	.5	1.0	.17	.17	.33	.33
.83	0	.67	.5	.5	1.0	.33	.33	.33	.83
.83	.33	0	.43	.5	1.0	.43	.17	.33	.83
.67	.5	.58	0	.7	1.0	.33	.33	.67	.83
.5	.5	.5	.3	0	.83	.17	.17	.33	.63
0	0	0	0	.17	0	0	0	0	.13
.83	.67	.57	.67	.83	1.0	0	.5	.77	.83
.83	.67	.83	.67	.83	1.0	.5	0	.67	.83
.67	.67	.67	.33	.67	1.0	.23	.33	0	.83
.67	.17	.17	.17	.37	.87	.17	.17	.17	0

GROUP THREE

0	0	0	0	0	0	0	0	0	0
1.0	0	.66	.56	.6	.6	.5	.5	.5	.5
1.0	.34	0	.5	.6	.6	.5	.5	.5	.4
1.0	.44	.5	0	.6	.6	.5	.5	.5	.5
1.0	.4	.4	.4	0	.6	.5	.5	.5	.4
1.0	.4	.4	.4	.4	0	.4	.4	.4	.4
1.0	.5	.5	.5	.5	.6	0	.5	.5	.5
1.0	.5	.5	.5	.5	.6	.5	0	.5	.5
1.0	.5	.5	.5	.5	.6	.5	.5	0	.5
1.0	.5	.6	.5	.6	.6	.5	.5	.5	0

GROUP FOUR

0	0	0	0	0	0	0	0	0	0
1.0	0	.5	.5	.5	.5	.54	.5	.5	.5
1.0	.5	0	.44	.5	.42	.54	.44	.46	.5
1.0	.5	.56	0	.5	.5	.52	.54	.5	.5
1.0	.5	.5	.5	0	.5	.54	.5	.5	.5
1.0	.5	.58	.5	.5	0	.54	.56	.5	.5
1.0	.46	.46	.48	.46	.46	0	.44	.5	.5
1.0	.5	.56	.46	.5	.44	.56	0	.5	.5
1.0	.5	.54	.5	.5	.5	.5	.5	0	.5
1.0	.5	.5	.5	.5	.5	.5	.5	.5	0

Figure 5: Final Average Group Matrices

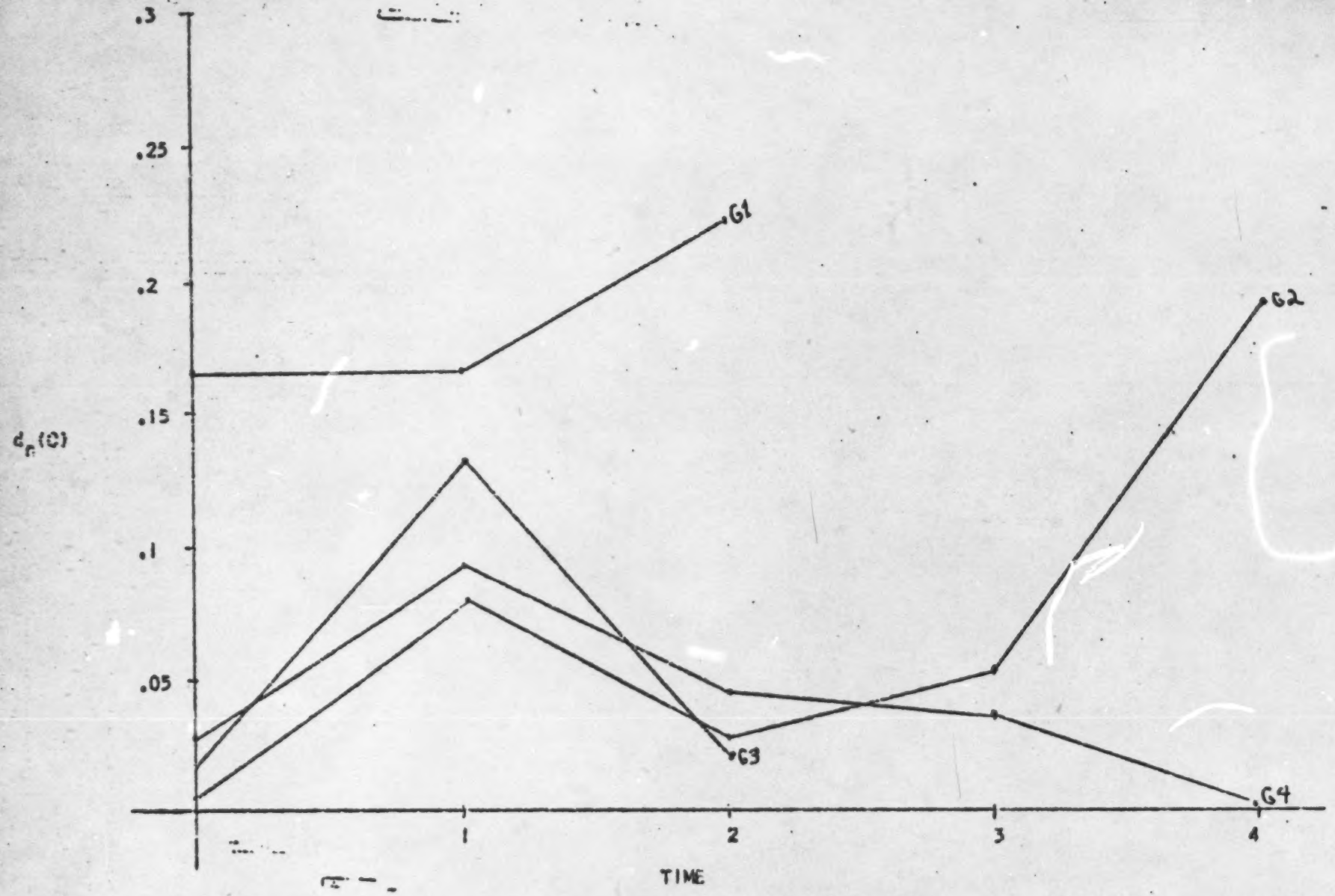


Figure 6: $d_n(C)$ for the Average Matrix for all Four Groups

TIME ZERO

	0	0	0	0	0	0	0	0	0	0
	1	0	0	0	0	1	0	1	0	0
	0	0	0	0	0	0	0	0	0	0
	0	0	0	0	0	0	0	0	0	0
	0	0	0	0	0	0	0	0	0	0
	0	0	0	0	0	0	0	0	0	0
	0	0	0	0	0	0	0	0	0	0
	0	0	0	0	0	0	0	0	0	0
	0	0	1	0	0	0	0	0	0	0
	0	0	1	0	0	0	0	0	0	0
COLUMN SUM	1	0	2	0	0	1	0	1	0	0

TIME ONE

	0	0	0	0	0	0	0	0	0	0
	1	0	1	0	0	1	0	1	0	0
	0	0	0	0	0	0	0	0	0	0
	1	0	1	0	0	0	0	0	0	0
	0	0	0	0	0	0	0	0	0	0
	0	0	0	0	0	0	0	0	0	0
	1	0	0	0	0	0	0	0	0	0
	0	0	0	0	0	0	0	0	0	0
	0	0	1	0	0	1	0	0	0	0
	0	0	1	0	0	0	0	0	0	0
COLUMN SUM	3	0	4	0	0	2	0	1	0	0

TIME TWO

	0	0	0	0	0	0	0	0	0	0
	1	0	0	0	0	0	0	0	0	0
	1	0	0	0	0	0	0	0	0	0
	1	0	0	0	0	0	0	0	0	0
	1	0	0	0	0	0	0	0	0	0
	1	0	0	0	0	0	0	0	0	0
	1	0	0	0	0	0	0	0	0	0
	1	0	0	0	0	0	0	0	0	0
	1	0	0	0	0	0	0	0	0	0
	1	0	0	0	0	0	0	0	0	0
COLUMN SUM	9	0	0	0	0	0	0	0	0	0

Figure 7: .9 α-level Sets for Group Three

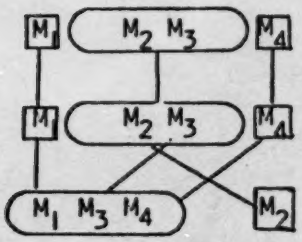
TIME	α-LEVEL	GROUP STRUCTURE
0	.4	
1	.4	
2	1.0	

Figure 8: Subgroup structure of Group One

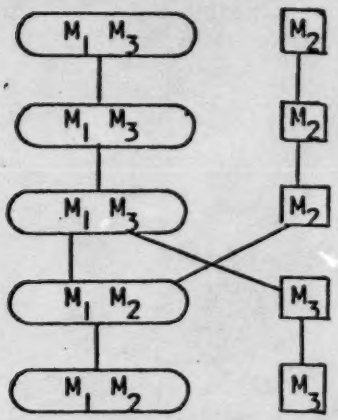
TIME	α-LEVEL	GROUP STRUCTURE
0	.5	
1	.4	
2	.4	
3	.5	
4	.5	

Figure 9: Subgroup Structure of Group Two

TIME	α-LEVEL
0	.5
1	1.0
2	1.0

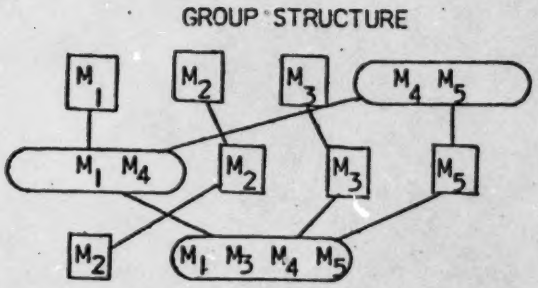


Figure 10: Subgroup Structure of Group Three

TIME	α-LEVEL
0	.5
1	.5
2	1.0
3	1.0
4	1.0

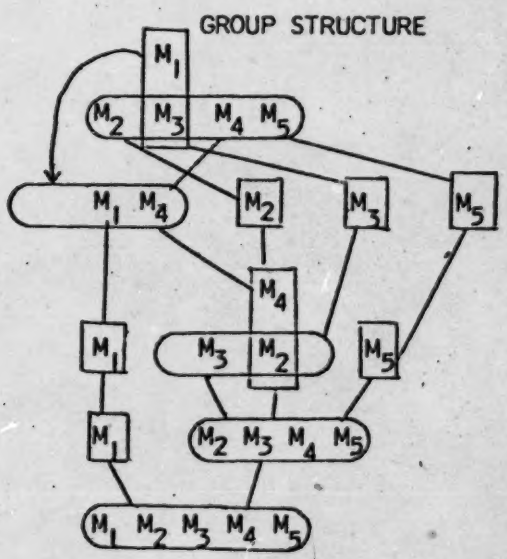


Figure 11: Subgroup Structure of Group Four

GROUP	MEMBER	TIME 0	1	2	3	4	PC
1	AVE	.4657	.4658	.435			18.5
	1	.2813	.3156	.3942			
	2	.4662	.4587	.4542			
	3	.2498	.2748	.4			
	4	.2867	.3289	.4			
2	AVE	.3984	.3696	.3906	.3781	.3249	18.67
	1	.2351	.3191	.3267	.3782	.4	
	2	.3258	.2436	.2231	.232	.176	
	3	.42	.3631	.3987	.3404	.0551	
3	AVE	.3953	.35	.3929			21.8
	1	.1164	.1689	.4			
	2	.3262	.3889	.2444			
	3	.3923	.3511	.4			
	4	.2111	.144	.4			
	5	.1813	.1444	.392			
4	AVE	.4086	.4382	.3809	.4138	.3986	21.6
	1	.3627	.3498	.3644	.3893	.3658	
	2	.252	.3773	.2818	.4098	.4	
	3	.2778	.3	.2333	.3911	.4	
	4	.3382	.3858	.2431	.4	.4	
	5	.4191	.3569	.3787	.4	.4	

Table 1: Fuzzy Measure for all Four Groups

	TIME				
GROUP	0	1	2	3	4
1	.16425	.1645	.21875		
2	.004	.076	.023	.054	.18775
3	.01175	.125	.01775		
4	.0215	.0955	.04775	.0345	.0035

Table 2: Normalized Distance from Consensus for all Four Groups